

second kind,  $Q_v^\mu(s)$ , when the order  $\mu$  is an integer  $m$ , the degree  $v$  is a half-odd integer  $n - 1/2$ , and the argument  $s$  exceeds unity.

The first table gives  $Q_{n-1/2}^m(s)$  to 11S for  $m = 0(1)5$ ,  $n$  varying from 0 through consecutive integers to a value (at least 29) for which the value of the function relative to that when  $n$  is zero is less than  $10^{-12}$ , and  $s = 1.1(0.1)10$ .

The second table differs from the first with respect to the argument, which here is  $\cosh \eta$ , where  $\eta = 0.1(0.1)3$ . As noted in the abstract and explained in the introduction, this form of the argument appears naturally in the solution of the potential problem in toroidal coordinates.

The last two tables consist of 16S values of  $Q_{n-1/2}^m(s)$  for  $n = 0$  and 1, for the same range of values of  $m$ ,  $s$ , and  $\eta$  as in the first two tables. These more extended decimal approximations were calculated independently by means of well-known formulas relating these toroidal functions to the complete elliptic integrals of the first and second kinds.

A useful introduction of 12 pages gives the derivation of these functions as solutions of Laplace's equation in toroidal coordinates, enumerates their principal properties, develops a continued fraction for the ratio of such functions of consecutive degree, and discusses the mathematical methods used in calculating the tables on IBM 1620 and IBM 7090 systems. Appended to the introduction is a list of five references.

Photographic offset printing of these tables from the computer sheets has not been completely satisfactory, as may be inferred from the two pages of corrigenda inserted to clarify a number of indistinctly printed tabular digits.

Despite such typographical imperfections, these extensive tables should prove generally useful to applied mathematicians.

J. W. W.

37[7].—M. KUMAR & G. K. DHAWAN, *Numerical Values of Certain Integrals Involving a Product of Two Bessel Functions*, Maulana Azad College of Technology, Bhopal, report and tables deposited in the UMT file.

In numerous applied problems, one encounters

$$I(\mu, \nu, \lambda) = \int_0^\infty e^{-pt^\lambda} J_\mu(at) J_\nu(bt) dt.$$

A discussion of this integral with references to tables is given by Luke [1]. Let  $a = t/h$ ;  $b = u/h$ ;  $u, t = 0.2(0.2)1.0$ ;  $p = 2$ ; and  $h = 1.05, 1.10, 1.30$  and  $1.50$ . For all possible combinations of these parameters the authors tabulate  $I(\mu, \nu, \lambda)$  to 6D for  $\mu = \nu = 0, 1, \lambda = 1, 2, 3$ , and for  $\mu = 1, \nu = 0$  and  $\lambda = 3$ . All the integrals can be expressed in terms of the complete elliptic integrals of the first and second kinds. These expressions are delineated in an introduction to the tables.

Y. L. L.

1. Y. L. LUKE, *Integrals of Bessel Functions*, McGraw-Hill Book Co., New York, 1962, pp. 314–318. (See also *Math. Comp.*, v. 17, 1963, pp. 318–320.)

38[9].—L. M. CHAWLA & S. A. SHAD, "On a trio-set of partition functions and their tables", Table, *J. Natur. Sci. and Math.*, v. 9, 1969, pp. 87–96.

The main table here lists  $q(n)$ ,  $r(n)$ ,  $\lambda(n)$ , and  $p(n)$  for  $n = 1(1)150$ . The last is the well-known partition function. The first is defined to be the number of partitions of  $n$  into *prime* parts. It is generated, of course, by

$$\sum_{n=0}^{\infty} q(n)x^n = [(1-x^2)(1-x^3)(1-x^5)(1-x^7)(1-x^{11})\dots]^{-1}.$$

Similarly,  $r(n)$  is the number of partitions of  $n$  into *composites* and *unity*. Finally,  $\lambda(n)$  is defined so as to take up the slack:

$$q(n) + r(n) + \lambda(n) = p(n).$$

A number of other notes in the same issue as this paper deal with these same functions and their generalizations.

The most interesting is  $q(n)$ , but this is not at all new. In [1] O. P. Gupta and S. Luthra give a longer table of this same function for  $n = 1(1)300$ . There is no reference to this earlier table here. The tables agree.

The obvious question is: How fast does  $q(n)$  grow? One sees at once that  $q(n)$  has a bit more than one-half the digits possessed by  $p(n)$ , and then that  $q(n)/\sqrt{p(n)}$  appears to grow slowly with  $n$ . If one now examines  $\log q(n)/\log p(n)$ , one finds that this ratio is about  $\frac{1}{2}$ ; it grows slowly, and reaches a maximum of 0.5572 at  $n = 120$ . Henceforth, the ratio very slowly decreases.

There is another function usually called  $q(n)$ , cf. [2]. Let us call it  $Q(n)$  here. This is the number of partitions into *odd* parts. One knows theoretically that

$$\log Q(n)/\log p(n) \sim 1/\sqrt{2} = 0.7071.$$

As Morris Newman pointed out to me, this is consistent with the foregoing, since there are fewer primes than odd numbers, and therefore  $Q(n)$  grows faster. As he also points out, the theory of  $q(n)$  was given by Hardy and Ramanujan [3]. This gives

$$\log q(n)/\log p(n) \sim (2/\log n)^{1/2}$$

and explains the slow decrease that occurs after  $n = 120$ . In fact, after  $n > e^8 \approx 3000$ ,  $q(n)/\sqrt{p(n)}$  will no longer increase, but decrease slowly to 0.

D. S.

1. O. P. GUPTA & S. LUTHRA, "Partition into primes," *Proc. Nat. Inst. Sci. India*, v. 21, 1955, pp. 181-184.

2. M. ABRAMOWITZ & I. A. STEGUN, editors, *Handbook of Mathematical Functions*, Dover, New York, 1965; Section 24, "Combinatorial analysis" (see 24.2.1, 24.2.2, Table 24.5).

3. G. H. HARDY & S. RAMANUJAN, "Asymptotic formulae for the distribution of integers of various types," *Proc. London Math. Soc.*, (2), v. 16, 1917, pp. 112-132; see Eq. (5.281).

**39[9].**—RICHARD B. LAKEIN & SIGEKATU KURODA, *Tables of Class Numbers  $h(-p)$  for Fields  $Q(\sqrt{-p})$* ,  $p \leq 465071$ , University of Maryland, College Park, Md., November 1965, copy deposited in the UMT file.

The main table, which consists of 76 Xeroxed computer sheets, contains the class numbers  $h(-p)$  for the first  $19 \cdot 2^{10} = 19456$  primes of the form  $4k + 3$ , the largest of which is 465071. This table therefore goes much further than those of Ordman [1] and Newman [2], which have already been reviewed, although they were computed well after the present table.